FILM FLOW OF A NONLINEAR VISCOELASTIC FLUID ALONG A CONICAL ROTOR

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Film flow of a nonlinear viscoelastic fluid, whose deformed behavior is described by using kinematic matrices, is considered along the surface of a rotating conical rotor.

Highly efficient centrifuge apparatuses in which drying and cooling by atomization, absorption, centrifuging, mixing, centrifugal casting, etc., are accomplished are used extensively in a number of branches of industrial production (chemical, petrochemical, etc.). Solutions and melts of polymers, suspensions and emulsions, colloidal solutions, dye materials, etc., treated in centrifuges exhibit a whole complex of nonlinear properties in the viscous-flowing state. Recently, a rheological equation of state of the type [1-5]

$$\sigma = -pJ + \mu_{ef}B_1 + \frac{1}{2}\beta_1 B_2 \tag{1}$$

has successfully been used to describe the anomaly in the viscous and elastic properties in solving engineering problems.

The kinematic matrices $B_{1,2}$ depend on the gradients of the velocities and accelerations and on higher derivatives of the velocity with respect to the time; μ_{ef} and β_1 are the coefficients of effective viscosity and normal stress, which are functions of the invariants $B_{1,2}$ and characterize the viscous and elastic properties of the material, respectively. The coefficient of effective viscosity can be written as [6]

$$\mu_{ef} = KE^{n-1}. \tag{2}$$

As has been shown in [7], the coefficient β_1 is proportional to the square of the viscosity and is determined by means of the formula

$$\beta_1 = \frac{1}{G_0}\mu_{ef}^2. \tag{3}$$

Let us consider an axisymmetric, steady stream of nonlinear viscoelastic fluid moving as a continuous laminar film over the surface of a rotating cone (Fig. 1). Let us consider the fluid motion in a special l, φ, δ coordinate system coupled rigidly to the cone. The system introduced is orthogonal.

We consider in the solution that: 1) the influence of film friction on the surrounding medium and of surface tension on the film flow is negligible; 2) the angular velocity of the fluid equals the angular velocity of the cone; 3) the fluid film thickness is considerably less than the corresponding coordinate l.

Taking account of the assumptions made above, and using the rheological equation of state (1), we obtain the differential equation of motion of the nonlinear viscoelastic fluid for this problem:

$$\rho v_l \frac{\partial v_l}{\partial l} + \rho v_l \frac{\partial v_l}{\partial \delta} = \rho F_l - \frac{\partial p}{\partial l} + K\frac{\partial}{\partial \delta}\left|\frac{\partial v_l}{\partial \delta}\right|^n - \frac{K^2}{G_0}\frac{\partial}{\partial l}\left|\frac{\partial v_l}{\partial \delta}\right|^{2n} - \frac{K^2}{lG_0}\left|\frac{\partial v_l}{\partial \delta}\right|^{2n}, \tag{4}$$

$$-\frac{\partial p}{\partial \delta} + \rho F_\delta = 0. \tag{5}$$

The quantities F_l and F_δ are expressed by the dependences

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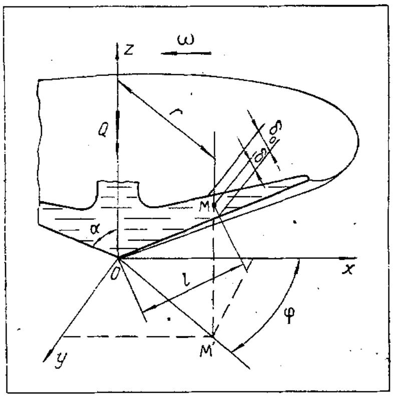


Fig. 1. Flow diagram.

$$F_l = \omega^2 r \sin\alpha - g\cos\alpha, \quad F_\delta = -(\omega^2 r\cos\alpha + g\sin\alpha).$$

Integration of (5) by using the boundary condition $p = p_0$ for $\delta = \delta_0$ yields

$$p = -(\delta_0 - \delta)\rho F_\delta + p_0. \tag{6}$$

According to the expression (6) obtained, by knowing the equation of the surface, the pressure at any point of the fluid film flowing over a rotating conical rotor can be found.

Let us estimate the order of the significance of $\partial p/\partial l$ by using (6):

$$\frac{\partial p}{\partial l} = \rho F_\delta \frac{\partial \delta_0}{\partial l} - (\delta_0 - \delta)\rho\frac{\partial F_\delta}{\partial l}.$$

Therefore, $(\partial p/\partial l) \approx \rho\omega^2 r\delta_0/l$, while the remaining terms in (4) are on the order of $\rho\omega^2 r$. Hence, $\partial p/\partial l$ can be discarded.

The boundary conditions for this problem are

$$\text{at } \delta = 0,\ v_l = 0;\ \text{at } \delta = \delta_0,\ \frac{\partial v_l}{\partial \delta} = 0,\ v_l = v_{l\max}. \tag{7}$$

Direct integration of (2) encounters definite mathematical difficulties. Hence, we use an approximate method of solution based on using integral relations [8]. To do this, we give the velocity profile in a form obtained analytically for a non-Newtonian fluid subject to a power law under analogous flow conditions [6]:

$$v_l = \frac{2n+1}{n+1}\,\frac{Q}{2\pi\delta_0 l\sin\alpha}\left[1 - \left(1 - \frac{\delta}{\delta_0}\right)^{\frac{n+1}{n}}\right]. \tag{8}$$

Let us multiply all the terms in (4) by r and let us integrate with respect to δ between 0 and δ_0:

$$\rho\int_0^{\delta_0} v_l\frac{\partial v_l}{\partial l}r\,d\delta + \rho\int_0^{\delta_0} v_l\frac{\partial v_l}{\partial \delta}r\,d\delta = \int_0^{\delta_0}\rho F_l r\,d\delta + K\int_0^{\delta_0}\frac{\partial}{\partial\delta}\left|\frac{\partial v_l}{\partial\delta}\right|^n r\,d\delta - \frac{K^2\sin\alpha}{G_0}\int_0^{\delta_0}\left|\frac{\partial v_l}{\partial\delta}\right|^{2n}d\delta - \frac{K^2}{G_0}\int_0^{\delta_0}\frac{\partial}{\partial l}\left|\frac{\partial v_l}{\partial\delta}\right|^{2n}r\,d\delta. \tag{9}$$

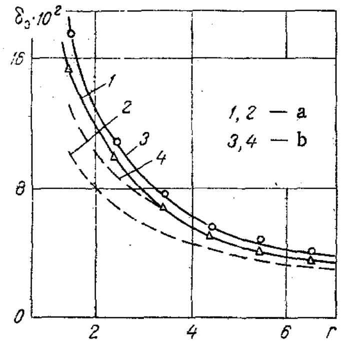
We consequently obtain

$$\begin{aligned}\frac{\partial\delta_0}{\partial r}&\left[\frac{K^2}{3G_0}\left(\frac{2n+1}{n}\,\frac{Q}{2\pi r}\right)^{2n}\frac{1-4n}{\delta_0^{4n+1}} - 2\rho\frac{2n+1}{3n+2}\left(\frac{Q}{2\pi r}\right)^2\frac{1}{\delta_0^3}\right] = \\ &= 2\rho\sin\alpha\frac{2n+1}{3n+2}\left(\frac{Q}{2\pi r}\right)^2\frac{1}{r\delta_0^2} - K\left(\frac{2n+1}{n}\,\frac{Q}{2\pi r}\right)^n\frac{1}{\delta_0^{2n+1}} + \\ &+ \rho F_l + \frac{K^2}{3G_0^2}\left(\frac{2n+1}{n}\,\frac{Q}{2\pi r}\right)^{2n}\frac{2(n-1)\sin\alpha}{r\delta_0^{4n}}.\end{aligned} \tag{10}$$

The differential equation (10) was solved further on an electronic computer by a numerical method involving the additional boundary condition

$$\text{at } r = r_{\text{init}},\ \delta_0 = \delta_{0\,\text{init}} \tag{11}$$

Here $\delta_{0\,\text{init}}$ takes account of the mechanical energy of the fluid delivered from outside, for example, the excess pressure in the nozzle. The value of $\delta_{0\,\text{init}}$ is determined experimentally.



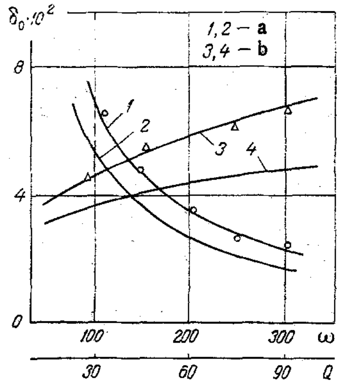


Fig. 2 Fig. 3

Fig. 2. Graphs of the dependence $\delta_0 = f(r)$ for $\omega = 104.7$ sec^{-1}, Q = 22 cm^3/sec, $\alpha = 60°$: a) 2.5% PAAM solution, n = 0.52, K = 42 dyn·sec/cm^2; $\rho = 1.039$ g/cm^3; 1) $G_0 = 27$ dyn/cm^2; 2) $G_0 = \infty$; b) 3% PAAM solution, n = 0.48, K = 70 dyn·sec/cm^2, $\rho = 1.045$ g/cm^3; 3) $G_0 = 40$ dyn/cm^2; 4) $G_0 = \infty$; r, cm. Points indicate experimental results; continuous curves are theoretical results.

Fig. 3. Dependence of film thickness of a 2.5% PAAM solution on the angular velocity of the rotor and the discharge: a — $\delta_0 = f(\omega)$, Q = 30 cm^3/sec; 1) $G_0 = 27$ dyn/cm^2; 2) $G_0 = \infty$; b — $\delta_0 = f(Q)$, $\omega = 157$ sec^{-1}; 3) $G_0 = 27$ dyn/cm^2; 4) $G_0 = \infty$; n = 0.52; K = 42 dyn·sec/cm^2; $\rho = 1.039$ g/cm^3; $\alpha = 90°$. δ_0, cm.

The expression (10) can be solved analytically also by giving the change in fluid-film thickness with respect to the radius $d\delta_0/dr$, as for non-Newtonian fluids [6]. In this case

$$\delta_0^{4n+1} + \frac{Q\delta^{4n-2}\sin\alpha}{2\pi^2 r^3 F_l}\left[\frac{2n+1}{3n+2}\delta_0 - \frac{n+1}{3n+2}\delta_*\right] - \frac{K}{\rho F_l}\left(\frac{2n+1}{n}\times\frac{Q}{2\pi r}\right)^n \delta_0^{2n} + \frac{K^2\sin\alpha}{3\rho r G_0 F_l}\left(\frac{2n+1}{n}\,\frac{Q}{2\pi r}\right)^{2n}\left[2(n-1)\delta_0 + \frac{(n+1)(1-4n)}{2n+1}\delta_*\right] = 0. \quad (12)$$

The solution of (12) affords the possibility of determining the fluid-film thickness as a function of the technological parameters Q, ω, and the properties of the reprocessed fluid. The maximum value of the discrepancy between the results of the numerical and analytical solutions is just 5%, which, in turn, indicates the legitimacy of the approximation made in deriving (12).

It should be noted that (10) for the power-law flow of a non-Newtonian fluid over a fixed disk ($G_0 = \infty$, $\omega = 0$) is

$$2\rho\,\frac{2n+1}{3n+2}\left(\frac{Q}{2\pi r}\right)^2\left[\frac{1}{\delta_0}\,\frac{\partial\delta_0}{\partial r} + \frac{\sin\alpha}{r}\right] = K\left(\frac{2n+1}{n}\,\frac{Q}{2\pi r}\right)^n\frac{1}{\delta_0^{2n-1}}, \quad (13)$$

and for a viscous inelastic fluid ($G_0 = \infty$, n = 1, $\omega = 0$) is

$$\frac{\partial\delta_0}{\partial r} + \frac{\delta_0\sin\alpha}{r} - \frac{5\pi\nu r}{Q} = 0. \quad (14)$$

The expression (14) has been obtained separately in [9]. Experiments were performed to verify the dependences (10) and (12) obtained. The test setup and method of performing the experiments are described in [10].

Used as model fluid were 2.5% and 3% aqueous solutions of polyacrylamide (PAAM). The rheological constants were determined by using a constant-pressure capillary viscometer.

As analyses of (12), the results of a numerical solution of (10), and the test data showed, the elastic properties of the fluid exert a strong influence on the flow hydrodynamics. The fluid-film thickness over the rotating cone increases (Fig. 2) with the increase in elasticity (with the decrease in the numerical value of G_0). Given for comparison in Fig. 2 are computed curves for non-Newtonian fluids without taking account of the elasticity, while the other conditions remain equal. It is seen from the graphs that the elasticity exerts quite a strong influence for small radii, but the discrepancies between the film thicknesses calculated taking and not taking account of elasticity diminish with the increase in the radius.

Other conditions being equal, fluid-film thicknesses grow with the increase in the discharge, where the growth intensity for nonlinear viscoelastic fluids is higher than for non-Newtonian fluids (Fig. 3). An increase in the number of rotor rotations results in a diminution in fluid-film thickness (Fig. 3).

The negligible deviation of the experimental values of the film thickness from the calculated values (3-5%) permits making a deduction about the validity of the approximations used in obtaining the dependences (10) and (12), as well as about the possibility of their application for practical computations of centrifugal machine components and chemical technological apparatuses.

NOTATION

p, hydrostatic pressure; J, unit matrix; K, n, rheological fluid constants; E, second invariant of the strain-rate tensor; G_0, initial modulus of high elasticity; σ, stress tensor; ρ, fluid density; l, φ, δ, cone generator, longitude, and distance between the axial line and a fluid particle M, respectively; r, distance between the surface and the axis of rotation; 2α, cone vertex angle; F_l, F_δ, projections of the mass forces in the l and δ directions; v_l, meridian velocity; δ_0, fluid-film thickness; ω, rotor angular velocity; Q, fluid discharge per second; ν, kinematic coefficient of viscosity; δ_*, fluid-film thickness calculated without taking account of the elastic properties.

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